



LONG-DISTANCE QUANTUM COMMUNICATION

CLAIM OF PRIORITY

This application claims the benefit of priority under 35 U.S.C. § 365 of International Patent Application Serial No. PCT/US02/15135, filed on May 20, 2002, designating the United States of America.

BACKGROUND OF THE INVENTION

Field of the Invention

The invention relates to long-distance quantum communication systems and methods for the scheme and method for entanglement-based quantum communication protocols, such as quantum teleportation and quantum cryptography.

Description of the Background

All current proposals for implementing quantum communication are based on photonic channels. The degree of entanglement generated between two distant quantum systems coupled by photonic channels decreases exponentially with the length of the connecting channel due to optical absorption and noise in the channel. This decrease represents a loss of "entanglement fidelity" in the communication system, which can be restored via "entanglement purification." Entanglement purification (referred to as simply "purification" in the literature) Purification means regaining a high degree of entanglement. See, e.g., Bennett, C. H. et al., "Purification of noisy entanglement and faithful teleportation via noisy channels", Phys. Rev. Lett. 76, 722-725 (1991). The Exponential exponential decay of entanglement as a function of channel length requires an exponentially increasing number of partially entangled states to obtain one highly entangled-states state.

The concept of quantum repeaters has been proposed to overcome the difficulty associated with the exponential fidelity decay of entanglement. See, e.g., Briegel, H.-J., Duer, W., Cirac, J. I. & Zoller, P., "Quantum repeaters: The role of imperfect local operations in quantum communication", Phys. Rev. Lett. 81, 5932-5935 (1991). A

quantum repeater using a cascaded entanglement purification protocol for a quantum communications communication systems- system ~~was is~~ proposed in the article by Knill, E., Laamme, R. & Zurek, W. H., entitled “-Resilient quantum computation;”, Science 279, 342-345 (1998), and in the article by Preskill, J., entitled “-Reliable quantum computers;”, Proc. R. Soc. Lond. A 454, 385-410 (1998). ~~This- The~~ proposed quantum repeater includes many short segments, with the length of each segment being comparable to a channel attenuation length. The proposed quantum repeater is used by first generating entanglement and purifying the entanglement for each segment; ~~the~~ The purified entanglement is then extended ~~to a longer length~~ by connecting two adjacent segments through entanglement swapping, and so on. Each entanglement swapping, however, decreases the overall entanglement, which therefore ~~requiring~~ requires a large number of iterations for sufficient purification.

~~The inventors recognized that, to implement the quantum repeater protocol, one needs to generate entanglement between distant quantum bits (qubits), store them for sufficiently long time and perform local collective operations on several of these qubits. The inventors recognized that the requirement of quantum memory is essential since all purification protocols are probabilistic. The inventors recognized that, when entanglement purification is performed for each segment of the channel, quantum memory can be used to keep the segment state if the purification succeeds and to repeat the purification for the segments only where the previous attempt failed. The inventors recognized that this is essentially important for polynomial scaling properties of the communication efficiency since no available memory would require that the purifications for all the segments succeed at the same time, and the probability of such event decreases exponentially with the channel length. The inventors recognized that the requirement of quantum memory implies that local qubits be stored in atomic internal states instead of the photonic states, since it is difficult to store photons. The inventors recognized that, if atoms are the local information carriers it would be difficult to implement quantum repeaters, since the prior art indicates that would require strong coupling between atoms and photons with high-finesse cavities for atomic entanglement generation, purification, and swapping. See also Cirac, J. I., Zoller, P., Kimble, H. J. & Mabuchi, H. Quantum~~

~~state transfer and entanglement distribution among distant nodes in a quantum network, Phys. Rev. Lett. 78, 3221-3224 (1997), Enk, S. J., Cirac, J. I. & Zoller, P. Photonic channels for quantum communication, Science 279, 205-207 (1998). Strong coupling technology does not appear to be feasible.~~

SUMMARY OF THE INVENTION

Implementing the long-distance quantum communication systems and methods of the present invention includes generating entanglement between distant quantum bits (qubits), storing them for sufficiently long time, and performing local collective operations on several of these qubits. Having quantum memory is essential for the system, since all purification protocols are probabilistic. When entanglement purification is performed for each segment of the channel, the quantum memory is used to keep the segment state if the purification succeeds, and to repeat the purification for only those segments where the previous attempt failed. Having quantum memory is important for the polynomial scaling of the communication efficiency because having no available memory means that the purifications for all the segments must succeed at the same time. Unfortunately, the probability of such this happening decreases exponentially with the channel length. The requirement of quantum memory implies that local qubits are stored in atomic internal states instead of the photonic states, since it is difficult (to the point of impracticality) to store photons. If atoms are used as the local information carriers, it is difficult to implement quantum repeaters because of the technically challenging proposition of having to use strong coupling between atoms and photons with high-finesse cavities for atomic entanglement generation, purification, and swapping.

An example embodiment of an apparatus of the present invention includes a light source, two cells each containing an atomic ensemble with a Lambda level configuration (examples are alkali atoms such as Sodium, Rubidium and Cesium), two filters, two channels, a beam splitter and two single-photon photodetectors.

~~It is an object of the invention to provide for entanglement generation with built-in entanglement purification between two atomic ensembles.~~

~~It is an object of the invention to provide for entanglement generation with built-in entanglement purification between multiple atomic ensembles.~~

~~It is another object of the invention to provide long quantum communication distance.~~

~~It is another object of the invention to provide for quantum teleportation, cryptography, and Bell inequality detection.~~

~~It is another object of the invention to provide a quantum communication system having built in entanglement purification and intrinsic fault tolerance.~~

~~It is another object of the invention to provide a quantum communication system in which resources scale polynomially with transmission distance.~~

~~It is another object of the invention to provide a quantum communication system relying upon collective excitations for entanglement of quantum systems.~~

~~The objects of the invention are provided by a system and method comprising generating pulses of light in a pulsed source of light, wherein each pulse of light includes at least one photon; propagating a first pulse of light generated in said pulsed source of light into a first ensemble having a first collective excitation state, wherein photons in said first pulse of light have an energy that can excite said first collective excitation; propagating a second pulse of light generated in said pulsed source of light into a second ensemble having a second collective excitation state, wherein photons in said second pulse of light have an energy that can excite said second collective excitation; receiving from said first ensemble and said second ensemble at an interferer for interfering light pulses substantially only photons resulting from generation of said first collective excitation and said second collective excitation; and receiving at a first single photon detector and second single photon detector light pulses from said interferer propagated to said interferer from said first ensemble and said second ensemble. Additional aspects of the~~

system and method include controlling photon detection of said first single photon detector and said second single photon detector with a photo detector controller; preventing photons not resulting from generation of collective excitations in one of said first ensemble and said second ensemble from reaching said interferer with a first filter disposed between said first ensemble and said interferer; preventing photons not resulting from generation of collective excitations from reaching said interferer with a second filter disposed between said second ensemble and said interferer; wherein an incident photon in a pulse of light generated by said source of light and transmitted into either said first ensemble or said second ensemble has a substantial probability of interacting with the ensemble to generate a collective excitation in the ensemble; wherein said ensemble comprises solid matter; wherein said ensemble comprises gaseous matter; wherein said ensemble comprises liquid matter; wherein said collective excitation of a photon in said pulses of light interacting with one of the ensembles to generate one of the collective excitations defines a Stokes process; wherein each of the ensembles have substantially identical collective excitation energies; wherein said first ensemble and said second ensemble comprise alkali atoms; wherein said first ensemble and said second ensemble comprise Cesium atoms; wherein a density of Cesium atoms in each one of said first ensemble and said second ensemble is between 1 and 100 atoms per cubic micro meter; wherein said preventing photons not resulting from generation of collective excitations in one of said first ensemble and said second ensemble from reaching said interferer comprises a wavelength sensitive first filter disposed between said first ensemble and said interferer; wherein said source comprises a synchronizer for synchronizing transmission from said source of two pulses; wherein said source comprises a laser; wherein said source comprises a flash lamp; wherein said first ensemble and said second ensemble contain molecules each of which has a collective excitation; ceasing generating pulses of light in said pulsed source of light when one of said detectors detects a pulse; entangling a third ensemble with a fourth ensemble, each having a collective excitation state, wherein photons in said first pulse of light have an energy that can excite said collective excitation state; entangling said first and second ensembles with said third and fourth ensembles; entangling comprises detecting a pulse propagated through a second beam splitter to one of a third single photon detector and a fourth single photon detector;

~~filtering pulses transmitted towards said second beam splitter; entangling said third and fourth ensembles with fifth and sixth ensembles; repeated applications of the basic steps for entanglement with additional sets of ensembles to provide long distance quantum communication in which resources only scale polynomially with the communication distance; repeated applications of the basic steps for entanglement with additional sets of ensembles to provide long distance entanglement generation comprising in which resources only scale polynomially with the communication distance.~~

~~In a preferred embodiment, objects of the invention are provided by an apparatus comprising a pulsed source of coherent or incoherent light, two cells with alkali (Lithium, Sodium, Potassium, Rubidium, Cesium, Francium) atoms gas samples (atomic ensembles), two filters, two channels, a beam splitter, two single photon photo detectors. Additional elements of the apparatus include the electronic control system for the detectors, and the system for synchronization of classical laser pulses. Methods of the invention use the embodiment of the apparatus summarized above to implement the objects of the invention. Importantly, the apparatus of the invention has communication efficiency that scales polynomially with channel length.~~

~~Additional physical~~ Physical systems that may be used for atomic ensembles of the invention include ensembles of atoms in similar potential energy traps, such as atoms in an ion trap, optical lattice position, or crystal lattice position, and which have collective excitations so that they may couple to photons, such as phonon excitations in solids. Preferably, these atomic systems have energy levels similar to the Lambda configuration level structure such that their collective excitations can be coupled to flying qubits in the same manner ~~ans~~ as for alkali metal atoms. Additionally, ensembles of localized excitation states such as electron states in superlattice structures including quantum dot ensembles, and Bose-Einstein condensates, may be useful ensembles for implementing the invention, and are also included in the definition of "atomic ensemble" as the phrase is used herein.

~~A first method uses preferred embodiment of the apparatus which is modified to extend quantum communication distance by means of entanglement swapping.~~

~~An Another embodiment for implementing a second method of the invention comprises a source, four atomic ensembles, two beam splitters, four single count photo detectors single-photon detectors (SPDs), and two phase shifts interferers. Additional elements comprise a synchronizer and single count photo detectors control system. Four atomic ensembles form two pairs of entangled atomic ensembles. A ~~second~~ Another method uses a preferred embodiment of the apparatus ~~which is modified to be efficiently used in the quantum communication protocols, such as quantum cryptography, and for Bell inequality detection.~~~~

~~An Another embodiment for implementing a third method of the invention comprises a source, six atomic ensembles, two beam splitters, and four single count photo detectors. Additional elements comprise a synchronizer and single count photo detectors control system. Six atomic ensembles form two pairs of entangled atomic ensembles and two pairs of atomic ensembles with collective atomic excitations. A ~~third~~ Another method uses a preferred embodiment of the apparatus ~~which is modified for realization of the quantum teleportation of the atomic polarization state.~~~~

~~This~~ The present invention relies upon collective rather than single particle excitations in atomic ensembles. It provides entanglement connection using simple linear optical operations, and it is inherently robust against the realistic imperfections. Here, ~~entanglement connection~~ “entanglement connection” means that two objects are connected, through entangled systems. Consider the system of FIG. 2 (described in more detail below) having, ~~given~~ two separate entangled bipartite systems. That is to say, the two pairs (L, I1) and (R, I2) where the subsystem L is entangled with subsystem I1 and subsystem R is entangled with subsystem I2. There is a way to create an entangled state between the two subsystems L and R. Such a process is generally called ~~entanglement connection~~ “entanglement connection.” A simple example of entanglement connection is called ~~entanglement swapping~~ “entanglement swapping.” Entanglement swapping is done

accomplished by performing a so-called Bell-measurement between the two other systems I1 and I2. A successful Bell-measurement ~~will connect~~connects the two subsystems L and R into an entangled state. In linear optics, it is often the case that a Bell-measurement can be successfully performed only probabilistically. A method to implement such a probabilistic Bell-measurement with linear optics and single-photon detectors of finite efficiency is shown in ~~our~~FIG. 2 and discussed below.

Quantum communication protocols of the present invention includes include quantum teleportation, cryptography, and Bell inequality detection. ~~Perfect entanglement~~ “Perfect entanglement” means that interacting physical systems are fully correlated. Increasing communication length in embodiments of the present invention only increases overhead in the communication time polynomially.

An example embodiment of the ~~The~~ present invention ~~can be~~ is implemented with only linear optical components such as beamsplitters, and ~~simple non-linear components such as~~ imperfect single-photon detectors. Yet, it allows intrinsic fault-tolerance. The invention is probabilistic. Many of the sources of errors do not affect the resulting state, even though they affect the probability of success. Some of the errors only ~~effect~~affect the coefficients of the successful state. Such modified coefficients can be easily compensated for or taken into account by system design without affecting polynomial scaling. Thus, one advantage of the present invention is that it allows for the distribution ~~of long distance~~ long-distance quantum communication or entanglement with resources that only scale polynomially with the communication distance.

The present invention also provides stable local information carriers, ~~i.e., i.e.,~~ stable quantum memory, due to the stability of collective excitations, resulting in relatively long decoherence times. In addition, relatively large cross sections exist between certain photons and certain collective excitations ~~use of which provides~~ that provide a substantial probability of collective excitation generation, per incident photon. These probabilities can be as great as approaching 1, preferably at least 0.5, and less preferably 0.25 or 0.1.

The inventors recognized that the light source may excite a number of other optical modes besides the desirable collective mode ~~to be~~ discussed below. The effects of ~~these~~ the other optical modes can be computed by standard perturbation theory in quantum mechanics. Therefore, it is simplest to understand the present invention by focusing on the desirable collective mode only. For this reason, a one-dimensional light propagation model and only the desirable collective mode are discussed herein below, and all other optical modes are ignored. However, the invention applies to more general configurations and is operable in the presence of other optical modes.

BRIEF DESCRIPTION OF THE FIGURES

Embodiments of the invention ~~will be~~ are described below in conjunction with the following ~~figures~~ Figures, in which like or corresponding elements are given the same identification reference number.

FIG. 1 is a schematic diagram of a novel apparatus of an example embodiment of an apparatus according to the present invention that is used to create entanglement between two atomic ensembles ~~the present invention;~~

FIG. 2 is a schematic diagram of a novel apparatus of another example embodiment of an apparatus according to the present invention that employs two pairs of atomic ensembles ~~for performing entanglement swapping of the invention;~~

FIG. 3 is a schematic diagram of another example embodiment of an novel apparatus of another embodiment according to the present invention that employs two pairs of atomic ensembles ~~for realizing quantum cryptography and Bell inequality detection; and~~

FIG. 4 ~~shows~~ is a schematic diagram setup of an apparatus of according to the present invention that is adapted ~~for performing probabilistic quantum teleportation of the an~~ atomic polarized state.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

In some formulas below, the "proportional to" sign (.about.) is replaced by a box symbol, such as "quadrature," which also means "proportional to."

An embodiment of an apparatus of the present invention comprises a light source, two cells each with a gas sample of atomic ensembles of atoms with a Lambda level configuration (examples are alkali atoms such as Sodium, Rubidium and Cesium), two filters, two channels, a beam splitter and two photodetectors, as shown in FIG. 1.

FIG. 1 shows is a schematic diagram of an example embodiment of an apparatus 1 according to the present invention. Apparatus 1 includes including a light source 10. Light source 10 may include a pulser or amplitude modulator to convert continuous light to pulsed light. Source 10 may also include electronics for synchronizing laser pulses. Light source 10 emits either continuous light or pulses of light. In an example embodiment, light source 10 is laser.

Apparatus 1 also includes, a source a light pulse synchronizer 20 optically coupled to light source 10 and from which emanates, a light beams (pulses) P1 and P2 along optical paths 30, and 40. Synchronizer 20 is adapted to receive light P0 from light source 10 and generate synchronized output beams (e.g., synchronized output pulses P1 and P2) along optical paths 30 and 40. Apparatus 1 also includes two atomic ensemble cells 50 and 60 (containing atomic ensembles L, R, respectively). Cells 50 and 60 are arranged along optical paths 30 and 40, respectively, so as to receive synchronized light pulses P1 and P2 from the synchronizer. In an example embodiment, atomic ensembles L and R emit Stokes light pulses SL and SR in response to being irradiated by the synchronized laser pulses. In an example embodiment, each cell contains an alkali gas sample containing about 10^{12} atoms of an alkali element. The cells may be, for example, glass cells with an interior antireflection coating. To facilitate enhanced coupling to optical pulses P1 and P2, the atomic ensembles are preferably optically thick along the direction of light propagation. This is achieved, for example, by providing a pencil-shaped cell. As mentioned above, the term "atomic ensemble" is not limited to atoms *per se*, and the

phrase as used herein is to be construed broadly to include other forms of matter capable of supporting a collective excitation and emitting Stokes light pulses, such as molecules, ensembles of localized excitation states such as electron states in superlattice structures including quantum dot ensembles, Bose-Einstein condensates, and the like.

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Apparatus 1 also includes filters 70, and 80 arranged downstream of and optically coupled to cells 50 and 60, respectively. Filters 70 and 80 are structured to select (i.e., transmit) a specified polarization and frequency corresponding to Stokes light pulses SL and SR emanating from cells 50 and 60, respectively, and to reject (i.e., reflect) all other light along optical paths 90 and 100, respectively.

Further included in apparatus 1 are ~~light beams 90, 100,~~ transmission channels 110 and 120 arranged downstream of and optically coupled to filters 70 and 80 via optical paths 130 and 140, respectively. In an example embodiment, transmission channels 110 and 120 are structures, such as optical fibers, through which propagate Stokes pulses SL and SR transmitted by filters 70 and 80.

Apparatus 1 further includes a beamsplitter 150 arranged at the intersection of transmission channels 110 and 120. Beam splitter 150 is preferably a mirror that provides 50% incident intensity reflection and transmission. In alternative embodiments, beam splitter 150 is or includes a 50:50 fiber coupler. Light exiting beamsplitter 150 can travel along two optical paths 160 and 170. Generally, beam splitter 150 is adapted to interfere the Stokes light pulses SL and SR from channels 110 and 120 with a 50:50 ratio. That is, the beam splitter serves as an “interferer” because it provides for interference of the two Stokes light pulses.

~~, stokes pulses 130, 140, beam splitter 150, stokes pulses 160, 170, single-photon photo detectors 180, 190, and photo-detector control system 200. Atomic ensembles 50, 60 each comprise a cell of an alkali gas sample. Source 10 comprises a coherent or incoherent light source.~~

~~Source 10 may include a pulser or amplitude modulator to convert continuous light to pulsed light. Source 10 may include electronics for synchronizing laser pulses. Source 10 emits either continuous light or pulses of light.~~

~~Cells 50, 60 each contain atomic ensembles L, R that can emit Stokes light pulses in response to illumination by synchronized laser pulses. Each cell contains an alkali gas sample containing about 10^{12} atoms of an alkali element. The cells are glass cells with antireflection inside coating. To facilitate enhanced coupling to light, the atomic medium is preferably optically thick along one direction. This can be easily achieved by working with a pencil shaped atomic sample.~~

~~Filters 70, 80 are structured to select (i.e., not reflect) specified polarization and frequency of light propagating from cells 50, 60, and to reject (i.e., reflect) non-selected polarization and frequency light propagating from cells 50, 60.~~

~~Transmission channels 110, 120 are structures, such as fiber optic, through which light transmitted through filters 70, 80 may propagate.~~

~~Beam splitter 150 is preferably a mirror that provides 50 percent incident intensity reflection and transmission. In alternative embodiments, beam splitter 150 may be replaced by a 50: 50 fiber coupler.~~

Apparatus 1 also includes single-photon photo-detectors (SPDs) 180, and 190, which in an example embodiment are conventional detectors SPDs capable of receiving and detecting light propagated along respective optical paths 160, and 170 from beam splitter 150.

~~Channels 110, 120 convey the Stokes light pulses from filters to beam splitter 150. Beam splitter 150 interferes the Stokes light pulses with a 50%-50% ratio.~~

~~That is, the beam splitter provides for interference of the two Stokes light pulses. Its function is therefore that of an interferer.~~

~~Single photon photo detectors 180, 190 detect pulses received from beam splitter 150.~~

~~Additional elements system of apparatus 1 include a photodetector control system 200 operably coupled to SPDs 180 and 190. Photodetector control system 200 includes, for example, electronics for controlling the operation of the single photon photo detectors 180, 190, SPDs. Apparatus 1 also includes ~~and~~ electronics (not shown), such as a controller, for controlling the operation of light source 10 and/or synchronizer 20 for synchronizing laser pulses P1 and P2 delivered to cells 50 and 60.~~

~~In operation, light source 10 generates synchronized laser pulses P0 and synchronizer 20 creates therefrom synchronized light pulses P1 and P2 that ~~propagating~~ propagate along optical paths 30, and 40 and, into atomic ensembles L and R contained in cells 50, and 60. Stokes light pulses SL and SR are emitted from the atomic ensembles L and R in cells 50, and 60, in response to ~~illumination being irradiated by the~~ synchronized laser pulses P1 and P2. For the generation of entanglement using Stokes light pulses generated in the cells, ~~the~~ light source 10 must operate at the wavelength ~~which~~ that matches the transition frequency of the ~~atoms of the corresponding alkali element which forms the gas~~ atomic ensembles sample in each the cell. Light Sources sources 10 capable of matching that can match the transition frequency of the alkali atoms atomic ensembles contained in cells 50, and 60 include certain coherent and partially coherent sources, such as certain lasers, including solid state, semiconductor, dye, free electron, and gas lasers.~~

~~-As mentioned above, in an In one preferred example embodiment, light source 10 is laser. More particularly, in an example embodiment, light source 10 is a diode laser emitting pulses P0 at a wavelength of 852 nm with a pulse duration of 0.1 microsecond and a typical repetition rate of at least 1 MHz. This wavelength matches the absorption energy of Cesium atom ensembles, at approximately room temperature, of 852 nm. "Approximately room temperature" in this context means temperatures between about 20~~

and about 30 Celsius. Preferably, each cell 50, and 60 contains ~~a cloud of~~ approximately N identical alkali atoms.

Preferably, the atomic ensembles L, and R in cells 50, and 60, respectively, comprise atoms having a Lambda-level structure. For instance, an example atomic ensemble includes Rubidium atoms in vapor form at around 70 to 90 degree Celsius, with atomic densities of about 10^{11} to 10^{12} per cubic centimeter, used in combination with a light source 10 in the form of ~~and~~ a diode laser with a frequency close to ~~its~~ the atomic level transition frequency for Rubidium. In one example embodiment, the alkali atom ensembles are at low temperature. For instance, in one embodiment, approximately 11 million sodium atoms are cooled to about 0.9 micro-Kelvin, just above the critical temperature for Bose-Einstein condensation. The, ~~the~~ cell defines the atomic ~~cloud ensemble~~ ensemble to have a length of about 350 ~~micro-meter~~ microns in the longitudinal direction as traveled by the light and about 55 ~~micro-meter~~ microns in the transverse directions, and a peak atomic density of about 11 atoms per cubic ~~micro-meter~~ micron. Generally, ~~Atomic~~ atomic densities in the range of 1 to 100 atoms per cubic centimeter ~~maybe useful~~ can be used.

A desirable longitudinal dimension for cells 50 and 60 of on the order of 3 cm ~~in the longitudinal direction of cell, 50, 60 is desirable~~. However, if a larger dimension is employed, ~~one can increase the overall decoupling strength between the source signal and the atom ensemble is increased~~. Depending on the identity and density of the ~~atom medium making up the atomic ensembles, employed~~, in some applications, a larger cell dimension of the cell may be desirable.

The relevant level structure of the atomic ensembles ~~atoms in the ensemble~~ contains $|g\rangle$ the ground state, $|s\rangle$, the metastable state for storing a qubit, and $|e\rangle$, the excited state. The transition $|g\rangle \rightarrow |e\rangle$ is coupled by the classical laser with the Rabi frequency, and the forward scattering Stokes light comes from the transition $|e\rangle \rightarrow |s\rangle$.
An off-resonant coupling with a large detuning Δ is assumed. The two atomic ensembles L and R are pencil shaped and illuminated by the synchronized classical laser

pulses P1 and P2. The forward-scattering Stokes pulses 130 and 140 are transmitted through polarization and frequency selective filters 70 and 80, propagate through channels 110 and 120 and interfere at 50%-50%50:50 beam splitter 150. Finally, single photons propagating through beam splitter 150 are detected in single photon detectors SPDs 180, and 190. If there is a click in either SPD detector 180 or 190, the process is finished and the entanglement is generated between the atomic ensembles L and R. Otherwise, a repumping pulse is first applied to the transition $|s\rangle \rightarrow |e\rangle$ on the ensembles L and R to set the state of the ensembles back to the ground state $|0\rangle_a^L \otimes |0\rangle_a^R$, then the same classical laser pulses as in the first round are applied to the transition $|g\rangle \rightarrow |e\rangle$ and the forward-scattering Stokes pulses are ~~detected again after the beam splitter generated~~. This process is repeated until finally there is a click in either the ~~detector SPD~~ SPD 180 or 190.

A pair of metastable lower states $|g\rangle$ and $|s\rangle$ corresponds ~~e.g.~~ e.g. to hyperfine or Zeeman sublevels of electronic ground state of alkali atoms. All the atoms are initially prepared in the ground state $|g\rangle$. A sample is illuminated by a short, off-resonant laser pulse that induces Raman transitions into the states $|s\rangle$. The forward-scattered Stokes light that is co-propagating with the laser is uniquely correlated with the excitation of the symmetric collective atomic mode S given by

$$S = (1/\sqrt{N_a}) \sum_i |g\rangle_i \langle s| ,$$

where the summation is taken over all the atoms (or molecules, etc.) in the atomic ensemble. In particular, an emission of the single Stokes photon in a forward direction results in the state of atomic ensemble given by

$$S^\dagger |0_a\rangle$$

where the ensemble ground state is given by

$$|0\rangle_a \equiv \otimes_i |g\rangle_i .$$

The light-atom interaction time t_d is short enough so that the mean photon number in the forward scattered Stokes pulse is much smaller than 1. An effective single-mode bosonic

operator a can be defined for this Stokes pulse with the corresponding vacuum state denoted by $|0\rangle_p$. The whole state of the atomic collective mode and the forward-scattering Stokes mode is written in the following form:

$$|\phi\rangle = |0\rangle_a |0\rangle_p + \sqrt{p_c} S^\dagger a^\dagger |0\rangle_a |0\rangle_p + o(p_c) \quad (1)$$

where p_c is the small excitation probability and $o(p_c)$ represents the terms with more excitations whose probabilities are equal or smaller than p_c^2 . A fraction of light is emitted in other directions due to the spontaneous emissions. However, whenever N_a is large, the contribution to the population in the symmetric collective mode from the spontaneous emissions is small. As a result there is a large signal-to-noise ratio for the processes involving the collective mode, which greatly enhances the efficiency of the present methods.

The methods of the present invention generate entanglement between two distant atomic ensembles L and R using ~~the configuration shown~~ apparatus 1 of in FIG. 1. Two laser pulses excite both atomic ensembles L and R simultaneously and the whole system is described by the state

$$|\phi\rangle_L \otimes |\phi\rangle_R, \text{ where } |\phi\rangle_L \text{ and } |\phi\rangle_R$$

are given by Equation (1) with all the operators and states distinguished by the subscript L or R. The forward scattered Stokes light (i.e., Stokes pulses 130 and 140) from both ensembles is combined at the beam splitter 150 and a ~~photodetector~~ click in either ~~detector SPD 180 or 190~~ measures the combined radiation from two samples,

$$a_+^\dagger a_+ \text{ or } a_-^\dagger a_- \text{ with}$$

$$a_\pm = (a_L \pm e^{i\varphi} a_R) / \sqrt{2}$$

Here, φ denotes an unknown difference of the phase shifts in the two-side channels 110 and 120. The variable φ has an imaginary part to account for the possible asymmetry of the setup, which ~~will also be~~ is corrected automatically. The asymmetry can be easily made very small, and φ is assumed real in the following. Conditional on the detector click, a_+ or a_- are applied to the whole state

$$|\phi\rangle_L \otimes |\phi\rangle_R$$

This provides the projected state of the atomic ensembles L and R being nearly maximally entangled with the form (neglecting the high-order terms $o(p_c)$):

$$|\Psi_\varphi\rangle_{LR}^\pm = (S_L^\dagger \pm e^{i\varphi} S_R^\dagger) / \sqrt{2} |0_a\rangle_L |0_a\rangle_R \quad (2)$$

The probability for getting a click is given by p_c for each round, so the process must be repeated about $1/p_c$ times for a successful entanglement preparation, and the average preparation time is given by

$$T_0 \sim t_\Delta / p_c$$

The states

$$|\Psi_\varphi\rangle_{LR}^+ \text{ and }$$

$$|\Psi_\varphi\rangle_{LR}^-$$

are easily transformed to each other by a local phase shift. Without loss of generality, it is assumed in the following that the entangled state

$$|\Psi_\varphi\rangle_{LR}^+$$

is generated.

The presence of the noise modifies the projected state of the ensembles to

$$\rho_{LR}(c_0, \varphi) = \frac{1}{c_0 + 1} \left(c_0 |0_a 0_a\rangle_{LR} \langle 0_a 0_a| + |\Psi_\varphi\rangle_{LR}^+ \langle \Psi_\varphi| \right) \quad (3)$$

where the "vacuum" coefficient c_0 is determined by the dark count rates of the photon detectors. It will be seen below that any state in the form of Equation (3) will be purified automatically to a maximally entangled state in the entanglement-based communication schemes. Therefore, this state is called an "effective maximally entangled" (EME) state with the vacuum coefficient c_0 determining the purification efficiency.

Each step of the process of entanglement generation and corresponding applications contains built-in entanglement purification, which makes the method resilient to realistic

noise and imperfections. In entanglement generation, the dominant noise is photon loss noise, which includes the contributions from the channel attenuation, the spontaneous emissions in the atomic ensembles (which results in the population of the collective atomic mode with the accompanying photon going to other directions), the coupling inefficiency of the Stokes light into and out of the channels, and the inefficiency of the ~~single-photon detectors~~ SPDs. The loss probability is denoted by $1 - \eta_p$ with the overall efficiency

$$\eta_p = \eta_p' e^{-L_0/L_{att}},$$

where the channel attenuation

$$e^{-L_0/L_{att}}$$

is separated ~~L_{att} is the channel attenuation length~~ from other noise contributions η_p' with η_p' independent of the communication distance L_0 (~~L_{att} is the channel attenuation length~~).

The photon loss decreases the success probability for getting a detector click from p_c to $\eta_p p_c$, but it has no influence on the resulting EME state. In this sense, the entanglement purification process is built-in and the process is intrinsically fault-tolerant. Due to this noise, the entanglement preparation time is replaced by

$$T_0 \sim t_\Delta / (\eta_p p_c).$$

In non-technical terms, many types of errors are in some sense, "self-correcting" in the process. Note that the method of the invention is probabilistic in that it depends on the existence-occurrence of a pulse being detected detection, where each outcome of an event is known to be either a "success" or a "failure," e.g., by the existence or non-existence of a pulse detection. Errors that are self-correcting affect the probability of success and the coefficients of the resulting states, but they do not, *a priori*, lead to a corrupted state, when the protocol indicates a "success" in the process. The change in coefficients in the process due to errors can be either pre-computed or observed in a real experiment. Therefore, those effects can be compensated or accounted for without affecting the main spirit of the protocol, e.g., the ability to communicate. The built-in nature of entanglement purification and, thus, the intrinsic fault-tolerant nature of the entanglement

purification process is discussed in more detail ~~in subsequent paragraphs below~~. In summary, the ~~process of the~~ present invention has built-in entanglement purification and is intrinsically fault-tolerant.

The second source of noise comes from the dark counts of ~~the single-photon detectors~~ SPDs 180 and 190. The dark count gives a detector click, but without population of the collective atomic mode, so it contributes to the vacuum coefficient in the EME state. If the dark count comes up with a probability p_c for the time interval t_d , the vacuum coefficient is given by

$$c_0 = p_{dc} / (\eta_p p_c),$$

which is typically much smaller than 1 ~~since~~ because the Raman transition rate is much larger than the dark count rate.

The final source of noise, which influences the fidelity to get the EME state, is caused by an event in which more than one atom is excited to the collective mode S whereas there is only one click in either ~~detector~~ SPD 180 or 190. The conditional probability for that event is given by p_c , so we can estimate the fidelity imperfection $\Delta F_0 \equiv 1 - F_0$ for the entanglement generation by

$$\Delta F_0 \sim p_c \quad (4)$$

By decreasing the excitation probability p_c , it is possible to make the fidelity imperfection closer and closer to zero with the price of a longer entanglement preparation time T_0 .

This is the basic idea of the entanglement purification. In this scheme, the confirmation of the click from the single-photon detector generates and purifies entanglement at the same time.

In ~~the~~ entanglement swapping, the dominant noise is loss, the losses, which include ~~the~~ such as associated with ~~contributions from the~~ SPD detector inefficiency, the

inefficiency of the excitation transfer from the collective atomic mode to the optical mode, and the small decay of the atomic excitation during the storage. By introducing the detector inefficiency, the imperfection is automatically taken into account by the fact that ~~detectors SPDs~~ 180 and 190 cannot distinguish between a single-single photon and two photons. The overall efficiency in the entanglement swapping is denoted by η_s . The loss in the entanglement swapping gives contributions to the vacuum coefficient in the connected EME state, since in the presence of loss a single detector click might result from two collective excitations in the ensembles I_1 and I_2 , and in this case, the collective modes in the atomic ensembles L and R have to be in a vacuum state. After taking into account the realistic noise, it is possible to specify the success probability and the new vacuum coefficient for the i^{th} entanglement connection by the recursion relations

$$p_i \equiv f_1(c_{i-1}) = \eta_s (1 - \frac{\eta_s}{2(c_{i-1} + 1)}) / (c_{i-1} + 1) \quad \text{and} \quad c_i \equiv f_2(c_{i-1}) = 2c_{i-1} + 1 - \eta_s. \text{The}$$

coefficient c_0 for the entanglement preparation is typically much smaller than $1 - \eta_s$,

$$c_i \approx (2^i - 1)(1 - \eta_s) = (L_i / L_0 - 1)(1 - \eta_s),$$

where L_i denotes the communication distance after i times entanglement connection.

With the expression for the c_i , the probability p_i and the communication time T_n are evaluated for establishing an EME state over the distance $L_n = 2^n L_0$. After the entanglement connection, the fidelity of the EME state also decreases, and after n times connection, the overall fidelity imperfection is given by:

$$\Delta F_n \sim 2^n \Delta F_0 \sim (L_n / L_0) \Delta F_0.$$

ΔF_n must be fixed to be small by decreasing the excitation probability p_c in Equation (4).

The entanglement connection scheme also has built-in entanglement purification function. This can be understood as follows: Each time entanglement is connected, the imperfections of the setup decrease the entanglement fraction $1/(c_i + 1)$ in the EME state. However, the entanglement fraction decays only linearly with the distance (the number of segments), which is in contrast to the exponential decay of the entanglement for the connection schemes without entanglement purification. The reason for the slow decay is that in each time of the entanglement connection, the protocol must be repeated until there is a detector click, and the confirmation of a click removes part of the added

vacuum noise since a larger vacuum components in the EME state results in more repetitions. The built-in entanglement purification in the connection scheme is essential for the polynomial scaling law of the communication efficiency.

Entanglement-based applications of the invention, such as long-distance quantum communication systems, also have built-in entanglement purification which makes them resilient to ~~the~~ realistic sources of noise. Firstly, the vacuum components in the EME states are removed from the confirmation of the detector clicks and thus have no influence on the fidelity of all the application schemes. Secondly, if the ~~single-photon detectors~~ SPDs and the atom-to-light excitation transitions in the application schemes are imperfect with the overall efficiency, then these imperfections only influence the efficiency to get the detector clicks with the success probability replaced by

$$p_a = \eta_a / [2(c_n + 1)^2] ,$$

and have no effects on the communication fidelity. Finally, the phase shifts in the stationary channels and the small asymmetry of the stationary setup are removed automatically when the EME state is projected onto the PME state, and thus have no influence on the communication fidelity. The noise not correctable by this method includes the detector dark count in the entanglement connection and the non-stationary channel noise and set asymmetries. The resulting fidelity imperfection from the dark count increases linearly with the number of segments L_n/L_0 , and from the non-stationary channel noise and set asymmetries increases by the random-walk law $(L_n/L_0)^{1/2}$. For each time of entanglement connection, the dark count probability is about 10^{-5} if a typical choice is made that the collective emission rate is about 10 MHz and the dark count rate is 10^2 Hz. This noise is negligible even if for communications over a long distance (~~10^3~~ the channel attenuation length L_{att} for instance). The non-stationary channel noise and setup asymmetries can also be safely neglected for such a distance. For instance, it is relatively easy to control the non-stationary asymmetries in local laser operations to values below 10^{-4} with the use of accurate polarization techniques for Zeeman sublevels.

In some formulas in this specification, the "proportional to" sign (\propto) is replaced by a box symbol, such as \propto , which also means "proportional to" herein.

Each of the entanglement generation and, connection, and application methods method of the present invention has built-in entanglement purification. As a result of this property, the communication fidelity can be fixed to be nearly perfect, and the time for preparing the system for communication increases only polynomially with the operating distance over which communication takes place. To communicate over a distance $L = L_n = 2^n L_0$, the overall fidelity imperfection must be fixed to be a desired small value ΔF_n , in which case the entanglement preparation time is

$$T_0 \sim t_\Delta / (\eta_p \Delta F_0) \sim (L_n / L_0) t_\Delta / (\eta_p \Delta F_n).$$

For an effective generation of the PME state, the total communication time

$$T_{tot} \sim T_n / p_a$$

with

$$T_n \sim T_0 \prod_{i=1}^n (1 / p_i).$$

The total communication time scales with the distance by the law

$$T_{tot} \sim 2 (L / L_0)^2 / (\eta_p \eta_a \Delta F_T \prod_{i=1}^n p_i) \quad (65)$$

where the success probabilities p_i, p_a for the i^{th} entanglement connection and for the entanglement application have been specified before. The expression (5) confirms that the communication time T_{tot} increases with the distance L only polynomially. In the first limiting case, the inefficiency $1 - \eta_s$ or the entanglement swapping is assumed to be negligibly small. It is deduced from Equation (5) that in this case the communication time

$$T_{tot} \sim T_{con} (L / L_0)^2 e^{L_0 / L_{am}},$$

with the constant

$$T_{con} = 2 t_\Delta / (\eta_p \eta_a \Delta F_T)$$

being independent of the segment and the total distances L_0 and L . The communication time T_{tot} increases with L quadratically. In the second case, it is assumed that the

inefficiency $1 - \eta_s$ is considerably large. The communication time in this case is approximated by

$$T_{tot} \sim T_{con} \left(L / L_0 \right)^{[\log_2(L/L_0)+1]/2 + \log_2(1/\eta_s - 1) + 2} e^{L_0 / L_{att}}$$

which increases with L still polynomially (or sub-exponentially in a more accurate language, but this makes no difference in practice since the factor $\log_2(L/L_0)$ is well bounded from above for any reasonably long distance). If T_{tot} increases with L/L_0 by the m^{th} power law $(L/L_0)^m$, there is an optimal choice of the segment length to be $L_0 = mL_{att}$ to minimize the time T_{tot} . As an estimation of the improvement in the communication efficiency, it is assumed that the total distance L is about $100L_{att}$, for a choice of the parameter $\eta_s \approx 2/3$, the communication time $T_{tot}/T_{con} \sim 10^6$ with the optimal segment length $L_0 \sim 5.7L_{att}$. This result is a dramatic improvement compared with the direct communication case, where the communication time T_{tot} for getting a PME state increases with the distance L by the exponential law

$$T_{tot} \sim T_{con} e^{L/L_{att}}$$

For the same distance $L_0 \sim 100L_{att}$, one needs $T_{tot}/T_{con} \sim 10^{43}$ for direct communication, which means that for this example the present scheme is 10^{37} times more efficient.

FIG. 2 shows is a schematic diagram of a quantum repeater apparatus system 201 similar to that of FIG. 1, but that includes two pairs of atomic ensembles. Apparatus 201 includes including source 10, and synchronizer 20. Apparatus 201 also includes, a first pair of optically coupled cells 230 and, 231-241 that contain respective of atomic ensembles I1, and I2L, and a second pair of optically coupled cells 231 and 242 that contain respective atomic ensembles I2 and R. Cells 230 and 231 are arranged downstream of and are optically coupled to synchronizer 20 via respective optical paths 250 and 251, cells 241, 242 of atomic ensembles L, R, light paths 250, 251, filters Filters 260, and 261 are arranged downstream of respective cells 230 and 231 and serve the same purpose as filters 70 and 80 in apparatus 1 of FIG. 1. Likewise, a beam splitter 270 is arranged at the intersection of optical paths 254 and 255 that are followed by Stokes pulses SI2 and SI1 that emanate from cells 231 and 230. Optical paths 254 and

255 optically couple filters 261 and 260 to respective SPDs 280 and 281. A photodetector control system 290 is coupled to SPDs 280 and 281 controls the operation of the SPDs, single photon detectors 280, 281, and photo detector control system 290. In addition, FIG. 2 shows unscattered light paths 252, 253, and stokes light paths 254, 255.

In the operation of apparatus 201, Concurrent light pulses P0 transmitted from emitted by source 10 pass through synchronizer 20, which creates synchronized light pulses P1 and P2 that traverse paths 250, and 251 to atomic ensembles I1, and I2 contained in respective in-cells 230, and 231. In atomic ensembles I1, and I2, the photons in the light pulse photons either undergo incident on the atomic ensembles either undergo a collective stokes scattering of from the atoms of the ensemble, or they do not. If they do not, they are reflected by filters 260, and 261 along optical paths 252, and 253 253. On the other had, if If they the photons are Stokes scattered, they are converted to a different frequency (and/or k vector) and are transmitted allowing by filters 260, and 261 to distinguish them from non-scattered light. The Stokes scattered light pulses are transmitted along optical paths 254, and 255 as Stokes pulses SI1 and SI2. The Stokes pulses (photons), and then interfere at beam splitter 270. Detectors SPDs 280, and 281 receive photons pulses propagating along respective optical paths 254, and 255, and each detector SPD generates a detection signal ("click") when it detects a pulse photon. Photo detector Photodetector control system 290 controls detectors 280, 290, and receives detection signals S280 and S281 from detectors SPDs 280, and 281 and processes these signals.

FIG. 2 shows two pairs of ensembles L, I1 and I2, R and four sites. Each of the atomic ensemble-pairs, i.e., pair L and, I1 and pair I2, and R, is prepared in an EME state in the form of Equation (3). The excitations in the collective modes of the atomic ensembles I1 and I2 are transferred simultaneously to the optical excitations by the repumping pulses applied to the atomic transition $|s\rangle \rightarrow |e\rangle$, and the The stimulated optical excitations, after a 50%-50% beam splitter, are detected by the single-photon detectors SPDs 180 and 190. If either detector SPD 180 or 190 generates a clicks click, the protocol is successful and an EME state is established between the atomic ensembles

L and R₂ thereby doubling the effective communication distance. If no ~~detector~~ SPD generates a ~~clicks~~ click, the previous entanglement generation and swapping process is repeated until ~~finally there is a click in detector~~ a click is generated by SPD 180 or 190, that is, until the protocol succeeds.

In an example embodiment of apparatus 201, ~~The~~ the two intermediated atomic ensembles I1 and I2 ~~can be~~ are replaced by one ensemble having two metastable states to store two different collective modes. The ~~50%-50%~~ beam splitter 270 operation ~~can be~~ is replaced by its functional equivalent, a so-called $\pi/2$ pulse. In the language of quantum field theory, let the collective modes be described by the two collective mode annihilation operators, a_1 and a_2 . Such a $\pi/2$ pulse maps a_1 and a_2 into their linear combinations in the following manner:

$$a_1 \text{ into } (a_1 + ia_2)/\sqrt{2} \text{ and } a_2 \text{ into } (a_1 - ia_2)/\sqrt{2} \text{ —————Eq. (A).}$$

Physically, such a $\pi/2$ pulse may be implemented by an external laser (not shown in FIG. 2) that emits a pulse onto the ~~the~~ ensemble containing the two said metastable states. The ensemble absorbs the $\pi/2$ laser pulse and changes its own state according to the transformation of the two said collective mode annihilation operators described by Eq. (A).

The embodiment in FIG. 2 provides two pairs of the entangled ensembles described by the state

$$\rho_{LI_1} \otimes \rho_{I_2R}, \text{ where}$$

ρ_{LI_1} and ρ_{I_2R} are given by Equation (3). In the ideal case, the ~~setup-system~~ shown in FIG. 2 measures the quantities corresponding to operators

$$S_{\pm}^{\dagger} S_{\pm} \text{ with } S_{\pm} = (S_{I_1} \pm S_{I_2})/\sqrt{2}.$$

If the measurement is successful (i.e., one of the detectors registers one photon), the ensembles L and R are prepared into another EME state.

The new φ -parameter is given by $\varphi_1 + \varphi_2$, where φ_1 and φ_2 denote the old φ -parameters for the two segment EME states. In the presence of the realistic noise and imperfections, an EME state is still created after a detector click. Noise only influences the success probability to get a click and the new vacuum coefficient in the EME state. In general the success probability p_1 is expressed and the new vacuum coefficient c_1 and

$$p_1 = f_1(c_0) \text{ and}$$

$$c_1 = f_2(c_0),$$

where the functions f_1 and f_2 depend on the particular noise properties.

The above method for connecting entanglement between atomic ensembles can be cascaded to arbitrarily extend the effective communication distance. For the i^{th} ($i = 1, 2, \dots, n$) entanglement connection, two pairs of atomic ensembles are first prepared in parallel ~~of ensembles~~ in the EME states with the same vacuum coefficient c_1 and the same communication length L_{i-1} , ~~and entanglement~~ Entanglement swapping is then performed as shown-described above in connection with in FIG. 2-is performed. This entanglement swapping attempt succeeds with a probability

$$p_i = f_1(c_{i-1}) .$$

After a successful detector click, the communication length is extended to $L_i = 2L_{i-1}$, and the vacuum coefficient in the connected EME state becomes

$$c_i = f_2(c_{i-1}) .$$

Since the i^{th} entanglement connection ~~need~~ needs to be repeated in average $1/p_i$ times, the total time needed to establish an EME state over the distance $L_n = 2^n L_0$ is given by

$$T_n = T_0 \prod_{i=1}^n (1/p_i) ,$$

where L_0 denotes the distance of each segment in the entanglement generation.

Entanglement can be attempted at a rate up to the pulse rate of light source 10. Typically available light sources operate up to the megahertz frequencies. In principle, pulse rates in the gigahertz frequencies are attainable, and would result in a sub-nanosecond time T_0 ~~of sub-nanosecond~~. Therefore, the average total time needed to establish entanglement in a system of a relatively large number of atomic ensembles using the systems and methods of the present invention can be shorter than a ~~micro-second~~ microsecond.

In ~~a second method~~ an example embodiment of the invention, the EME is used in the communication protocols, such as quantum teleportation, cryptography, and Bell inequality detection. ~~FIG. 3 shows the schematic setup for the realization of quantum cryptography.~~

FIG. 3 is a schematic diagram of another example embodiment of an entanglement-based apparatus schematically shows system 301 that employs two pairs of atomic ensembles L1 and R1 and L2 and R2. Apparatus 301 can be used, for example, in a QKD system or for testing Bell inequalities. Apparatus 301 including includes source 10, and synchronizer 20, as described above, with cells 330, and 331 arranged downstream of and optically coupled to synchronizer 20. Cells 330 and 331, containing contain atomic ensembles L1, and R1, respectively. Apparatus 301 further includes cells 340, and 341, which respectively contain-containing atomic ensembles L2, and R2, optically coupled to respective cells 300 and 331. Further, cell 330 and 340 are optically coupled via optical path 342, while cells 331 and 341 are optically coupled via optical path 344. A first beam splitters-splitter 350, is arranged at the intersection of optical paths 353 and 354 leading from cells 330 and 340 to SPDs 370 and 371, respectively. A second beam splitter 351, is arranged at the intersection of optical paths 357 and 358 leading from cells 331 and 341 to SPDs 380 and 381, respectively. A first phase shifter phase shifters-360, is-361 arranged in optical path 353 between cell 340 and first beam splitter 350. A second phase shifter 361 is arranged in optical path 357 between cell 341 and second beam splitter 361. , photo-detectors 380, 381, photo-detectors 390, 391, and photo-Photo detector control system 390290 is operably coupled to SPDs 370, 371, 380 and 381.-

FIG. 3 shows an embodiment in long-distance quantum key distribution or in testing of Bell inequalities. In FIG. 3, two pairs of atom ensembles (L1, R1) and (L2, R2) are shown. In apparatus 301, ~~Each each~~ pair of atomic ensembles --, for example, pair (L1, R1)-- can be the result of a concatenated application of apparatus 201 of FIG. 2 employing ~~(entanglement swapping and entanglement purification,)~~ as discussed in preceding paragraphs above. Additional laser sources 10 and ~~any~~ synchronization systems 20 ~~are probably~~ may be needed to set up the initial state ~~of~~ for atomic ensemble pairs (L1, R1) and also (L2, R2). ~~However, since these additional elements have been discussed in preceding paragraphs concerning FIG. 2, we will leave them out from~~ In apparatus 301 of FIG. 3, it is ~~and assume~~ assumed that the preparation of the initial states of atomic ensemble pairs (L1, R1) and (L2, R2) has already been performed. Formally, the state of the atomic ensemble pair, say (L1, R1), is described as an EME state defined earlier. The arrow 385 indicates ~~the fact that the~~ atomic ensemble pair (L1, R1) has been prepared in an EME state and is, ~~thus,~~ quantum mechanically correlated with entanglement. Similarly, the arrow 386 indicates that ~~the fact that the~~ atomic ensemble pair (L2, R2) has been prepared an EME state and is, ~~thus,~~ quantum mechanically correlated with entanglement.

In FIG. 3, each atomic ensemble pair (L1, R1) may be separated spatially by a very long distance, say 100, 1000, or even 10,000 km. However, atomic ensembles L1 and L2 are supposed to be spatially located much closer than the distance between atomic ensembles L1 and R1. A typical distance between atomic ensembles L1 and L2 may be 10 m or less, although a much longer distance of say one km may also be acceptable. Similarly, atomic ensemble R1 and R2 are supposed to be spatially located much closer than the distance between atomic ensembles L1 and R1. Two pairs of ~~atom-atomic~~ ensembles are needed because a pair of photons ~~are~~ is needed for an experiment in quantum key distribution, whereas a pair of ~~atom-atomic~~ ensemble typically ~~gives out~~ generates only a single photon. The ~~beam splitters~~ beam splitters 350 and 351 allow ~~us to entangle for~~ entanglement to occur between the two pairs of ~~atom-atomic~~ ensembles, thus generating a maximally entangled state between them.

In ~~our~~ an example preferred embodiment, the ~~detector/control~~ photodetector control system ~~290 consists of~~ includes two separate detector/control subsystems (not shown) located near ~~to~~ atomic ensemble pairs (L1,L2) and (R1,R2), respectively. The detector/control subsystem ~~that is located near to~~ atomic ensemble pair (L1,L2) controls the setting of ~~the~~ phase shifter 360 and, thus, decides ~~on~~ the type of measurement to be performed on ~~the systems~~ atomic ensembles (L1,L2). Similarly, the detector/control subsystem ~~that is located near to~~ (R1,R2) controls the setting of ~~the~~ phase shifter 361 and decides on the type of measurement to be performed on the ~~systems~~ atomic ensembles (R1,R2). The output ~~to the experiment is given by~~ of apparatus 301 is governed by how many and which of the ~~single-photon photo-detectors~~ SPDs 370, 371, 380 and 381 have detected a photon.

As ~~schematically~~ shown in FIG. 3, one optical ~~pulse~~ path 342 traverses in sequence atomic ensembles L1 and L2, ~~while another~~. ~~Another~~ optical ~~pulse~~ path 344 traverses atomic ensembles R1 and R2. Photons generated by the ~~desired~~ collective excitations in either atomic ensemble L1 or L2 propagate to beam splitter (interferer) 350. The relative angles shown between photon of optical paths 342 and 344 into ensembles and the relative angles of optical paths 353, 354 and 357 and 358 photon paths from ensembles to beam splitters 350, 351 are drawn for convenience of illustration. For example, the photons generated by collective excitations in an atomic ensemble may propagate in the same direction as incident photons, and subsequently be diverted by a wavelength-dependent reflector disposed on a far side of the ensemble. Detection at ~~detector~~ SPD 370 or 371 indicates a collective excitation in atomic ensemble pair (L1,L2), which indicates that atomic ensembles L2 and R2 ~~is are~~ entangled with atomic ensembles L1 and R1.

A more detailed and more mathematical description of FIG. 3 is as follows.:

~~Two~~ The pairs of atomic ensembles, ~~pair~~ (L1, R1) and ~~pair~~ (L2, R2) (or alternatively two pairs of metastable states) are each prepared in EME states. The collective atomic excitations on each side are transferred to the optical excitations, which, respectively after

a relative phase shift φ_L or φ_R , and a 50%-50% beam splitter, are detected by the single-photon detectors

$$D_1^L, D_2^L \text{ and}$$

$$D_1^R, D_2^R.$$

There are four possible coincidences of

$$D_1^L, D_2^L \text{ with } D_1^R, D_2^R,$$

which are functions of the phase difference $\varphi_L - \varphi_R$. Depending on the choice φ_L and φ_R , this setup can realize both the quantum cryptography and the Bell inequality detection.

It is not obvious that the EME state (3), which is entangled in the Fock basis, is useful for these tasks since in the Fock basis it is experimentally hard to ~~do~~ perform certain single-bit operations. EME states can be used to realize all these protocols with simple experimental configurations.

The state of the two pairs of ensembles is expressed as

$$\rho_{L_1 R_1} \otimes \rho_{L_2 R_2}, \text{ where}$$

$$\rho_{L_i R_i} (i = 1, 2)$$

denote the same EME state with the vacuum coefficient c_n if entanglement connection has been done n times. The φ -parameters in

$$\rho_{L_i R_i} (i = 1, 2)$$

are the same provided that the two states are established over the same stationary channels. Only the coincidences of the two-side detectors are registered, so the protocol is successful only if there is a click on each side. Under this condition, the vacuum components in the EME states, together with the state components

$$S_{L_1}^\dagger S_{L_2}^\dagger |vac\rangle \text{ and } S_{R_1}^\dagger S_{R_2}^\dagger |vac\rangle, \text{ where}$$

$$|vac\rangle \text{ denotes the ensemble state}$$

$$|0_a 0_a 0_a 0_a\rangle_{L_1 R_1 L_2 R_2},$$

have no contributions to the experimental results. So, for the measurement ~~cheme~~
~~apparatus of shown by~~ FIG. 3, the ensemble state

$$\rho_{L_1 R_1} \otimes \rho_{L_2 R_2}$$

is effectively equivalent to the following "polarization" maximally entangled (PME) state

$$|\psi\rangle_{PME} = (S_{L_1}^\dagger S_{R_2}^\dagger + S_{L_2}^\dagger S_{R_1}^\dagger) / \sqrt{2} |vac\rangle \quad (6)$$

The success probability for the projection from

$$\rho_{L_1 R_1} \otimes \rho_{L_2 R_2} \text{ to } |\psi\rangle_{PME}$$

(i.e., the probability to get a click on each side) is given by

$$p_a = 1 / [2(c_n + 1)^2]$$

In FIG. 3, the phase shift

$$\psi_\Lambda (\Lambda = L \text{ or } R)$$

together with the corresponding beam splitter operation are equivalent to a single-bit rotation in the basis

$$\{|0\rangle_\Lambda \equiv S_{\Lambda_1}^\dagger |0_a 0_a\rangle_{\Lambda_1 \Lambda_2}, |1\rangle_\Lambda \equiv S_{\Lambda_2}^\dagger |0_a 0_a\rangle_{\Lambda_1 \Lambda_2}\}$$

with the rotation angle $\theta = \psi_\Lambda / 2$. Now, there is the PME state and it is possible to perform the desired single-bit rotations in the corresponding basis, and thus to implement quantum cryptography and Bell inequality detection. For instance, to distribute a quantum key between the two remote sides, we simply choose -randomly from the set $\{0,$

$\pi/2\}$ with an equal probability, and keep the measurement results (to be 0 if D_1^L clicks,

and 1 if D_1^R clicks) on both sides as the shared secret key if the two sides become aware that they have chosen the same phase shift after the public declare. This is exactly the Ekert scheme and its absolute security is already proven. For the Bell inequality detection, we infer the correlations

$\bar{E}(\psi_L, \psi_R) \equiv P_{D_1^L D_1^R} + P_{D_2^L D_2^R} - P_{D_1^L D_2^R} - P_{D_2^L D_1^R} = \cos(\psi_L - \psi_R)$ from the measurement of the coincidences

$P_{D_1^L D_1^R}$, etc.

For the setup shown in apparatus of FIG. 3, the output is

$$|E(0, \pi/4) + E(\pi/2, \pi/4) + E(\pi/2, 3\pi/4) - E(0, 3\pi/4)| = 2\sqrt{2}$$

whereas for any local hidden variable theories this value should be below 2.

A third method of using the preferred embodiment realizes the setup for probabilistic quantum teleportation of the atomic "polarization" state. Similarly to the second method, two pairs of ensembles L_1, R_1 and L_2, R_2 are prepared in the EME states. An atomic "polarization" state

$$(d_0 S_{I_1}^\dagger + d_1 S_{I_2}^\dagger) |0_a 0_a\rangle_{I_1 I_2}$$

can be teleported with unknown coefficients d_0, d_1 from the left to the right side, where

$$S_{I_1}^\dagger, S_{I_2}^\dagger$$

denote the collective atomic operators for the two ensembles I_1 and I_2 (or two metastable states in the same ensemble). The collective atomic excitations in the ensembles I_1, L_1 and I_2, L_2 are transferred to the optical excitations, which, after a 50%-50% beam splitter, are detected by the single-photon detectors

$$D_1^I, D_1^L \text{ and } D_2^I, D_2^L.$$

If there are a click in D_1^I or D_1^L and a click in D_2^I or D_2^L , the protocol is successful. A φ -phase rotation is then performed on the collective mode of the ensemble R_2 conditional

on that the two clicks appear in the detectors D_1^L, D_2^L or D_1^R, D_2^R . The collective excitation in the ensembles R_1 and R_2 , if appearing, would be found in the same "polarization" state

$$(d_0 S_{R_1}^\dagger + d_1 S_{R_2}^\dagger) |0_a 0_a\rangle_{R_1 R_2}$$

The established long-distance EME states can be also used for faithful transfer of unknown quantum states through quantum teleportation, with the ~~setup shown~~apparatus of FIG. 4.

FIG. 4 ~~schematically shows~~is a schematic diagram of another example embodiment of an entanglement-based apparatus system 401 according to the present invention that includes two pairs of atomic ensembles (L1, R1) and (L2, R2). Apparatus 401 includes including a secondary control system 495 that includes light source 10 and synchronizer 20. Secondary control system 495 is operably coupled to photodetector control system 490 via a communication link 496. Secondary control system 495 is also operably coupled to cells 430 and 431 that contain atomic ensembles R1 and R2, respectively, via respective optical pulse channels 497, and 498. Apparatus 401 also includes, cells 430, 431, cells 440, and 441 that respectively contain atomic ensembles L1 and L2 and that are optically coupled to cells 430 and 431, respectively, as indicated by arrows 485 and 486.

Apparatus 401 further includes, and cells 442, and 443 that contain respective atomic ensembles I1 and I2. Optical paths 550 and 552 optically couple cells 440 and 442 to a first, beam splitters 450, 451 beam splitter (interferer) 450 and then to SPDs 470 and 471. Likewise optical paths 551 and 553 optically couple cells 441 and 443 to a second beam splitter (interferer) 451 and then to SPDs 480 and 481, single photon detectors 470, 471, and 480, 481, and photo detector control system 490.

In operation of the apparatus 401 of FIG. 4, embodiment, photodetector photo detector control system 490 receives detection signals ("clicks") from single photon detectors SPDs 470, and 471, and 480, and 481. Based upon that data the received detection signals,, photo detector photodetector control system 490 sends optical pulse generation control data in the form of signal S490 via communication link 496 to secondary control system 495. Based upon that the optical pulse generation control data, secondary control system 495 controls an optical source's the generation of optical pulses P1 and P2 for transmission to atomic ensemble R1 or atomic ensemble R2. The two-

headed arrow denoting optical paths 485 and 486, respectively, between ensembles R1,L1 indicate that ~~those two ensembles are initially entangled~~. The two headed arrow ~~between ensembles R2,L2 indicate that those two ensembles are initially entangled~~. atomic ensembles R1 and L1 are entangled, and that atomic R2 and L2 are entangled.

Apparatus 401 of FIG. 4 shows an embodiment can be used for probabilistic quantum teleportation. Consider the atomic atom ensembles (L1, R1) and (L2,R2). ~~shown in FIG. 4.~~ Each atomic ensemble pair; ~~for example, (L1,R1) --~~ can be the result of a concatenated application of apparatus 201 of FIG. 2, which employs ~~(entanglement swapping and entanglement purification, as)~~ discussed in preceding paragraphs above. Additional laser sources 10 and any synchronization systems 20 ~~are probably~~ may be needed to set up the initial state of atomic ensemble pairs (L1,R1) and also (L2,R2). However, ~~since those additional elements have been discussed in preceding paragraphs concerning FIG. 2, we will leave them out from FIG. 4 and assume~~ it is assumed that the preparation of the initial states of atomic ensemble pairs (L1,R1) and (L2,R2) has already been performed. Formally, the state of a atomic ensemble pair, say (L1, R1), is described as an EME state defined earlier. The double Arrow arrow for optical path 485 indicates the fact that the atomic ensemble pair (L1,R1) has been prepared in an EME state and is, thus, quantum mechanically correlated with entanglement. Similarly, the double arrow for optical path 486 indicates that the fact that the atomic ensemble pair (L2,R2) has been prepared an EME state and is; thus, quantum mechanically correlated with entanglement.

In apparatus 401, FIG. 4, each atomic ensemble pair (L1,R1) may be separated spatially by a very long distance, say 100, 1000, or even 10,000 km. However, atomic ensembles L1 and L2 are supposed to be spatially located much closer than the distance between atomic ensembles L1 and R1. A typical distance between atomic ensembles L1 and L2 may be 10 meters or less, although a much longer distance of say one km may also be acceptable. Similarly, atomic ensembles R1 and R2 are supposed to be spatially located much closer than the distance between atomic ensembles L1 and R1. Note that two pairs of ~~atom~~ atomic ensembles are needed because the experiment concerns a state of a pair of

polarization entangled photons, whereas a pair of ~~atom~~-atomic ensembles typically gives out only a single photon.

In apparatus 401, FIG. 4, the pair of ~~atom~~-atomic ensembles (I1,I2) is in some unknown quantum state. The goal of ~~FIG. 4~~apparatus 401 is to teleport this unknown state from atomic ensemble pair (I1,I2) to the ~~right hand~~remote atomic ensemble pair ~~side~~-(R1,R2). Here, atomic ensemble I1 is located spatially close to atomic ensemble L1, and atomic ensemble I2 is spatially close to L2. A typical distance between atomic ensembles I1 and L1 may be 10 meters or less, although a much longer distance of say one km may also be acceptable. The same is true Similarly, for atomic ensembles I2 and L2. Here, a photodetector ~~/~~ control system 490 can be used for controlling ~~the photo detectors~~SPDs 470, 471, 480 and 481. Effectively, atomic ensemble pair (L1,L2) constitutes a logical qubit (call it qubit A) and atomic ensemble pair (I1,I2) constitutes an unknown input qubit (call it qubit E). In ~~FIG. 4~~ apparatus 401, a probabilistic Bell-measurement is performed on the joint system of ~~the above said~~ qubits (A,E). The atomic ensemble pair (R1,R2) represents a third qubit, ~~call it~~ qubit B. Photodetector ~~Control~~ control system 490 obtains the outcome of the probabilistic Bell-measurement and sends it via the communication link 496 via signal S490 to the secondary control system 495. Conditional on the information in the outcome signal it has received, ~~the~~ secondary control system 495 may apply one ~~out of a plurality of~~ of a number of possible operations on the atomic ensemble systems, R1 and R2, via ~~the control lines,~~optical paths 497 and 498, to regenerate the state of teleported qubit (qubit E). In other words, after the regeneration, the state of the qubit B, which is stored in the physical system of the atomic ensemble pair (R1,R2), now becomes the state of the original qubit E. Therefore, the quantum signal has been sent over long distances of ~~potentially~~ 100, 1000 or even 10000 km.

~~The~~ A mathematical and more detailed description of apparatus 401 of FIG. 4 is as follows:

In ~~this setup~~ apparatus 401, if two ~~detectors click~~ SPDs click -on the left side (i.e., SPDs 471 and 480), there is a significant probability that there is no collective excitation on the right side of the apparatus, since the product of the EME states

$$\rho_{L_1 R_1} \otimes \rho_{L_2 R_2}$$

contains vacuum components. However, if there is a collective excitation appearing from the right side, its "polarization" state would be exactly the same as the one input from the left. The teleportation here is probabilistic and needs posterior confirmation; but if it succeeds, the teleportation fidelity would be nearly perfect since in this case the entanglement is equivalently described by the PME state (6). The success probability for the teleportation is also given by

$$p_a = 1 / \left[2 (c_n + 1)^2 \right],$$

which determines the average number of repetitions for a successful teleportation.

The foregoing describes the theory of the invention and practical applications based upon collective excitation of ~~a correlated system of alkali atoms~~ atomic ensembles, and provides some specific examples of ~~systems for using the invention for long-distance signal communication systems~~. However, ~~the inventors recognize that~~ collective excitations in many other types of correlated systems exist, and that many of those excitations systems may be found suitable for use as a correlated system of the invention instead of the atomic ensembles noted in the foregoing examples. Thus, while only certain features of the present invention have been outlined and described herein, many modifications and variations will be apparent to those skilled in the art. Therefore, the claims appended hereto are intended to cover all such modifications and equivalents that fall within the broad scope of the invention.